S. S. College. Jehanabad (Magadh University)

Department : Physics

Subject : Quantum Mechanics

Class : B.Sc(H) Physics Part III

Topic: Spin Eigenfunction and Stern Gerlach Experiment

Teacher : M. K. Singh

Spin eigenfunctions & Pauli spin matrices

The spin eigen function are given for spin up ($m_s = +\frac{1}{2}$) and spin down ($m_s = -\frac{1}{2}$) is:

$$\chi_+ = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \ \& \ \chi_- = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Hence the complete wave function based on n, l, m_l, m_s is given by:

$$\psi_{n l m_l m_s}(r, \pm) = R_{nl} Y_{l m_l}(\theta, \phi)\, \chi_{\pm}$$

The spin angular momentum operator is given in matrix form:

$$\vec{S} = \frac{1}{2}\hbar\vec{\sigma}$$

where $\vec{\sigma}$ is called Pauli spin operator, It components have the matrix

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

These are called Pauli spin matrices.

- The determinant of Pauli spin matrices

$$|\sigma_x| = |\sigma_y| = |\sigma_z| = -1$$

Magnetic Moment due to Spin

The electron due to orbital angular momentum
It is given by :

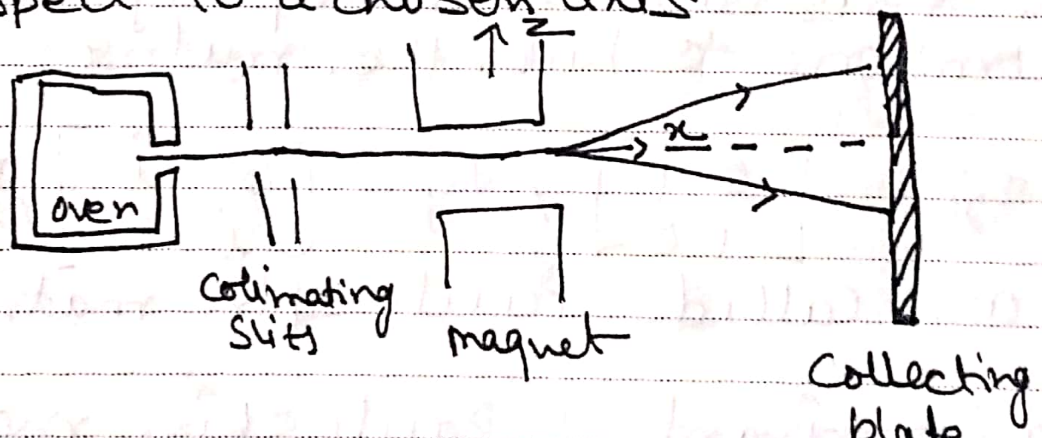
$$\vec{\mu}_L = -\frac{e}{2m}\vec{L}$$

The assumption has proved to be correct.
The magnetic moment due to spin is given by :-

$$\vec{\mu}_S = -g_S \frac{e}{2m}\vec{S}$$

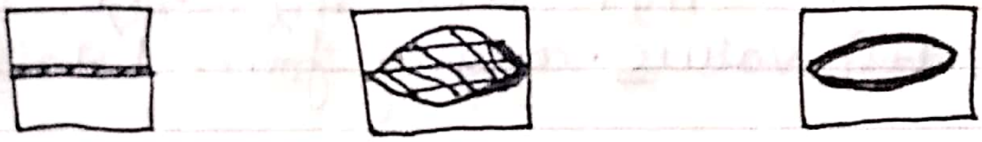
The Stern-gerlach experiment

The space quantization of angular momentum is explained by stern-gerlach experiment.
The property that the angular momentum vector associated with a quantum state can take up only certain specified directions in space with respect to a chosen axis.



: The stern-Gerlach apparatus

- When no magnetic field is applied, the beam produces a single trace on the collecting plate.
- When the field is switched on the trace splits distinctly into more than one trace.



a) No magnet field b) expected Pattern (c) Actual Pattern

If $\vec{\mu}$ is the resultant magnetic moment of the atom and $\vec{B}$ is applied magnetic field then Potential energy, $U = -\vec{\mu}\cdot\vec{B} = \left(\frac{e}{2m}\right)\vec{L}\cdot\vec{B}$

Since field is in $+z$ direction; $B = B_z$

$$U = \frac{e}{2m} L_z B_z$$

and force on the atomic dipole

$$F_z = -\frac{du}{dz} = -\frac{e}{2m} L_z \frac{dB_z}{dz}$$

- If quantization of angular momentum did not exist, $\vec{L}$ for different atom could be pointing in different direction.
- The L_z can take any value from $+L$ to $-L$ and when the magnetic field is turned on different atom will move up or down by different amount. and we get result like pattern (b).

Stern-gerlach found that instead of it beam split into two subbeams, each of which made a well defined separate trace on the plate. It is only possible if the angular momentum is quantized. when L is quantized then

$L_z \rightarrow m_l \hbar$ and $m_l = -l, 0, +l$

i,e $(2l+1)$ values ranging from $-l$ to l.

However, there was a problem. Since $(2l+1)$ is odd and it would imply that the number of traces on the plate would alway be odd but this is not the case for many atoms.

Example:- silver, hydrogen, lithium, sodium, potassium, copper and gold.

This is due to the quantization of spin quantum number. Hence the subbeams are observed for it. The stern-Gerlach experiment was a direct confirmation of space quantization and the concept of electron spin.